hep-th/0310075
October 2003

Vacuum Polarization in an Anti-de Sitter Space as an Origin for a Cosmological Constant in a Brane World

Li-Xin Li

Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, USA

lli@cfa.harvard.edu

Abstract

In this Letter we show that the vacuum polarization of quantum fields in an anti-de Sitter space naturally gives rise to a small but nonzero cosmological constant in a brane world living in it. To explain the extremely small ratio of mass density in the cosmological constant to the Planck mass density in our universe ($\approx 10^{-123}$) as suggested by cosmological observations, all we need is a four-dimensional brane world (our universe) living in a five-dimensional anti-de Sitter space with a curvature radius $r_0 \sim 10^{-3}$cm and a fundamental Planck energy $M_{\mathrm{P}} \sim 10^9$GeV, and a scalar field with a mass $m \sim r_0^{-1} \sim 10^{-2}$eV. Probing gravity down to a scale $\sim 10^{-3}$cm, which is attainable in the near future, will provide a test of the model.

Much attention has recently been paid to the idea that our universe is a four-dimensional brane (three dimensions of space plus one dimension of time) embedded in a five-dimensional bulk universe. This fact is mainly caused by the discoveries that by confining the standard model particles on a brane the extra dimensions can be larger than previously anticipated then the very large hierarchy between the electroweak and the Planck energy scales is relaxed [2, 1], and that on a brane embedded in a five-dimensional anti-de Sitter space Newton's gravitational law can be recovered on distances larger than submillimeter as required by the gravity experiments that have been taken so far [3, 4, 5]. A lot of work has since then been done on brane cosmology, and people have shown that in the case when the bulk universe is empty away from the brane the cosmology on the brane deviates from that given by the standard general relativity only in the epoch before the cosmic nucleosynthesis when the energy density in the universe is sufficiently high ([6, 7, 8, 9, 10, 11, 12, 13] and references therein). Observational test of brane models by supernovae has also been discussed [14].

Despite the remarkable success of the brane model, the cosmological constant problem remains unsolved [13]. In the brane scenario, it is usually assumed that the tension of the brane exactly cancels the effect of the negative cosmological constant in the bulk universe, giving rise to a zero cosmological constant in the brane universe. However, observations of gravitational lensing, high redshift type Ia supernova, cosmic microwave background, and cluster counts consistently show that the universe is in all likelyhood flat and accelerating, with the present mass density being composed of about 70% of cosmological constant (or dark energy), 30% of dark and ordinary matter [15, 16, 17, 18, 19, 20, 21]. In Planck units $\hbar = c = 1$, the cosmological constant is extremely small: $\rho_\Lambda/\rho_{\rm P} \approx 10^{-123}$, where ρ_Λ is the mass density in the cosmological constant, $\rho_{\rm P}$ is the Planck mass density in our four-dimensional universe. The existence of such a small but nonzero cosmological constant is usually called the cosmological constant problem and continues to be a big mystery for modern cosmology and theories of elementary particles [22, 23, 24]. For current efforts toward solving this problem, see [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36] and references therein.

In this Letter we propose an explanation for the existence of a small but nonzero cosmological constant in the brane world scenario. For simplicity, we assume that the bulk spacetime is a five-dimensional anti-de Sitter space. We show that the vacuum polarization of a massive scalar field in the bulk spacetime has a stress-energy tensor that behaves like a small positive cosmological constant. As in the standard scenario of a brane world we assume that the brane tension cancels the effect of the bare cosmological constant in the anti-de Sitter space, then on the brane a small but nonzero cosmological constant exists, which we will show is consistent with cosmological observations if the mass of the scalar field is $\sim r_0^{-1}$ where r_0 is the curvature radius of the anti-de Sitter space.

An anti-de Sitter space is a spacetime that has the maximal symmetry and a negative constant curvature, supported by a negative cosmological constant [37, 38]. For an N-dimensional anti-de Sitter space, its curvature radius is related to the cosmological constant by $r_0 = [(N-1)(N-2)/(-2\Lambda)]^{1/2}$. The Riemann curvature tensor is $R_{abcd} = 2\Lambda(g_{ac}\,g_{bd} - g_{ad}\,g_{bc})/[(N-1)(N-2)]$. The corresponding Ricci tensor $R_{ab} = 2\Lambda g_{ab}/(N-2)$ satisfies the N-dimensional Einstein equation $R_{ab} - (1/2)\,R\,g_{ab} + \Lambda\,g_{ab} = 8\pi M_{\rm P}^{2-N}\,T_{ab}$ when the N-dimensional stress-energy tensor $T_{ab} = 0$, where $R = g^{ab}R_{ab} = 2\Lambda N/(N-2)$, $M_{\rm P}$ is the

N-dimensional Planck mass.

Quantum fields in an anti-de Sitter space have been studied extensively [39, 40, 41, 42, 43, 44, 45, 46, 38]. For a quantum state that is maximally symmetric (i.e., anti-de Sitter-invariant), the two-point function $G(x,x')$ depends only on the geodesic distance $\mu(x,x')$ between x and x'. For a massive scalar field $\phi(x)$ in an N-dimensional anti-de Sitter space of radius r_0, the two-point function for a maximally symmetric vacuum state, $G(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle$, is given by [42]

$$G(x,x') = G(\mu) = A\cosh^{-2\alpha}\left(\frac{\mu}{2r_0}\right) F\left(\alpha, \alpha - \frac{N}{2}+1; 2\alpha - N + 2; \cosh^{-2}\frac{\mu}{2r_0}\right) , \qquad (1)$$

where

$$\alpha \equiv \frac{1}{2}\left[N - 1 + \sqrt{(N-1)^2 + 4m^2 r_0^2}\right] , \qquad A \equiv \frac{\Gamma(\alpha)\, r_0^{2-N}}{2^{2\alpha+1}\pi^{(N-1)/2}\Gamma\left(\alpha - \frac{N}{2} + \frac{3}{2}\right)} . \qquad (2)$$

Here F denotes the hypergeometric function, Γ denotes the Gamma function, and m is the mass of the scalar field particle.

The two-point function given by Eq. (1) satisfies the following two conditions: (a) falls off as fast as possible at spatial infinity $\mu^2 \to \infty$, and (b) has the same strength $\mu^2 \to 0$ singularity as in a flat space. It is worth to note that the two-point function so defined can be obtained from the continuation of a Euclidean Green function on H^N—an N-dimensional hyperbolic Euclidean space, but cannot be obtained from the continuation of a Euclidean Green function on S^N—an N-dimensional sphere [41, 43].

For $N < 2$ the limit $\mu \to 0$ in Eq. (1) is finite, and we have

$$G(x,x) = \frac{\Gamma\left(1 - \frac{1}{2}N\right)\Gamma(\alpha)}{2^N \pi^{N/2}\Gamma(\alpha - N + 2)}\, r_0^{2-N} . \qquad (3)$$

$G(x,x)$ is analytic in the complex N-plane apart from simple poles. Therefore, it may be extended throughout the whole complex N-plane [47] (see also [41, 45]). Especially, when N is odd, the $G(x,x)$ given by Eq. (3) is finite and can be taken to be the regularized value for $\phi(x)^2$, according to the dimensional regularization procedure [48].

For a vacuum state that is maximally symmetric the expectation value of its stress-energy tensor must be given by $\langle T_{ab}\rangle = T\, g_{ab}/N$, where $T = g^{ab}\langle T_{ab}\rangle$ is the trace [48]. For a massive scalar field whose $G(x,x)$ is given by Eq. (3), which is finite and constant when N is odd, we have $T = -m^2\langle\phi(x)^2\rangle = -m^2 G(x,x)$. Then we have

$$\langle T_{ab}\rangle = -\frac{\Gamma\left(1 - \frac{1}{2}N\right)\Gamma(\alpha)}{N 2^N \pi^{N/2}\Gamma(\alpha - N + 2)}\, m^2 r_0^{2-N} g_{ab} , \qquad (4)$$

which is the renormalized stress-energy tensor when N is odd [46].[1] The stress-energy tensor in Eq. (4), which describes the vacuum polarization effect of a massive scalar field in an odd

[1]In this Letter we focus on the case when N is odd since we have assumed the bulk space has five dimensions. When N is even, further care must be taken to remove the poles in the Gamma function, see [47, 48, 41, 45].

N-dimensional anti-de Sitter space, corresponds to a cosmological constant

$$\Lambda' = \frac{\Gamma\left(1-\frac{1}{2}N\right)\Gamma(\alpha)}{N 2^{N-3}\pi^{(N-2)/2}\Gamma\left(\alpha-N+2\right)}\, m^2 \left(M_{\rm P} r_0\right)^{2-N} \ . \tag{5}$$

When $N=5$—the case that is of particular interest in this Letter, we have

$$\Lambda' = \frac{m^2}{15\pi M_{\rm P}^3 r_0^3}\left(3+m^2 r_0^2\right)\sqrt{4+m^2 r_0^2}\ , \tag{6}$$

which is positive when $m^2 r_0^2 > 0$ or $-4 < m^2 r_0^2 < -3$, negative when $-3 < m^2 r_0^2 < 0$, zero when $m^2 r_0^2 = 0, -3$, or -4.

The gravitational equation on a brane in the five-dimensional anti-de Sitter space is then (see, e.g., [49])

$$\begin{aligned} {}^{(4)}R_{ab} - \frac{1}{2}\,{}^{(4)}R\,{}^{(4)}g_{ab} + \frac{1}{2}\left(\Lambda+\Lambda'\right)\,{}^{(4)}g_{ab} \ \ &= \ \ -\frac{16\pi^2}{3}M_{\rm P}^{-6}\sigma^2\,{}^{(4)}g_{ab} + \frac{32\pi^2}{3}M_{\rm P}^{-6}\sigma\,{}^{(4)}T_{ab} \\ &\quad + [\text{higher order terms}]\ , \end{aligned} \tag{7}$$

where the index "(4)" on the left shoulder denote a quantity defined on the four-dimensional brane world, σ is the tension of the brane, ${}^{(4)}T_{ab}$ is the stress-energy tensor of matter (with the tension term subtracted) on the brane, and "higher order terms" stands for terms quadratic in ${}^{(4)}T_{ab}$ that are important only in sufficiently high energy regime. Following the standard procedure in the brane world scenario, let us set

$$-\frac{16\pi^2}{3}M_{\rm P}^{-6}\sigma^2 = \frac{1}{2}\Lambda\ , \qquad \frac{32\pi^2}{3}M_{\rm P}^{-6}\sigma = 8\pi m_{\rm P}^{-2}\ , \tag{8}$$

where $m_{\rm P} = G^{-1/2}$ is the Planck mass in the four-dimensional space. Then, the standard Einstein equation (without a cosmological constant) is recovered if $\Lambda' = 0$ and higher orders terms are neglected. By the second equation of (8), σ must be positive, i.e. the brane must have a positive tension.

If $\Lambda' \neq 0$ and is given by Eq. (6), then there is a cosmological constant in the brane world, whose value is

$${}^{(4)}\Lambda = \frac{m^2}{30\pi M_{\rm P}^3 r_0^3}\left(3+m^2 r_0^2\right)\sqrt{4+m^2 r_0^2}\ . \tag{9}$$

Of course, one can argue that Eq. (8) can be modified to cancel Λ' then the cosmological constant in the brane world will be zero. However, doing so would introduce more fine tune to the theory as Eq. (8) is already a fine tune. We think the true question is, for reasonable values of parameters, if the ${}^{(4)}\Lambda$ given by Eq. (9) is consistent with the value of a cosmological constant that has been implied by the cosmological observations. In the rest of the Letter we will show that the answer to this question is "Yes".

By construction, there are two fundamental energy scales in the five-dimensional anti-de Sitter bulk space: $M_{\rm P}$ and r_0^{-1}. The four-dimensional Planck mass $m_{\rm P}$ and the brane tension σ are derived from $M_{\rm P}$ and r_0^{-1} by Eq. (8):

$$m_{\rm P}^2 = M_{\rm P}^3 r_0 \,, \qquad \sigma = \frac{3}{4\pi} M_{\rm P}^3 r_0^{-1} \,, \tag{10}$$

where $\Lambda = -6/r_0^2$ has been used. The requirement to recover Newton's gravitational law down to scales of the submillimeter order (see [50, 51]) puts a stringent constraint on r_0: $r_0 < 10^{-2}$cm, which by Eq. (10) implies that $M_{\rm P} > 5.4\times 10^{-11} m_{\rm P} \approx 6.7\times 10^8$GeV and $\sigma > \left(2.8\times 10^{-16} m_{\rm P}\right)^4 \approx (3000{\rm GeV})^4$.

The four-dimensional cosmological constant ${}^{(4)}\Lambda$ depends on another parameter: the mass of the scalar field, m, which in principle should be constructed from $M_{\rm P}$ and/or r_0^{-1}. Since $M_{\rm P} \gg r_0^{-1}$ [otherwise Eq. (10) would imply $m_{\rm P} \lesssim M_{\rm P}$], a natural possibility would be $m \sim r_0^{-1}$, not $m \sim M_{\rm P}$.

An elegant argument for a discrete mass spectrum for a scalar field in an anti-de Sitter space was given by Allen and Jacobson [42]. In an anti-de Sitter space, which has topology $S^1 \times \mathbb{R}^{N-1}$ and contains closed timelike curves, the two-point function should be invariant under the transformation $\mu \to \mu + i\,2\pi r_0 j$,[2] where $2\pi r_0$ is the timelike circumference at the "neck" of the anti-de Sitter space, j is any integer. From Eq. (1) we have

$$G(\mu + i\,2\pi r_0 j) = \exp(-i\,2\pi j\alpha)\, G(\mu) \,, \tag{11}$$

then α must be an integer so that $G(\mu + i\,2\pi r_0 j) = G(\mu)$. Let $\alpha = n+4$, where n is an integer, then from Eq. (2) we have

$$m^2 = \frac{n(n+4)}{r_0^2} \,, \tag{12}$$

where we have set $N = 5$. When $n = 0, -1, -2, -3$, or -4, $m^2 \leq 0$ and from Eq. (9) we have ${}^{(4)}\Lambda = 0$. When $n \geq 1$ or $n \leq -5$, $m^2 > 0$ and from Eq. (9) we have ${}^{(4)}\Lambda > 0$. Since $m^2 < 0$ leads to the existence of tachyons which are usually thought unphysical, we restrict our treatment to the case of $m^2 \geq 0$ and assume $n = 0, 1, 2, ...$ in Eq. (12). [Since Eq. (12) is invariant under the transformation $n \to -n-4$, $n = -4, -5, -6, ...$ give same values of m^2 as $n = 0, 1, 2, ...$] The above argument supports the assumption that r_0^{-1} is the natural scale for the mass m.

Of course, one can avoid closed timelike curves by "unwrapping" the S^1 dimension, or equivalently, considering the universal covering space of an anti-de Sitter space with topology $\mathbb{R}^N$. Then, the mass spectrum of the scalar field will be continuous and Eq. (11) gives an explicit expression for $G(x, x')$ when x and x' are separated by j sheets [42]. However, we point out that in the brane world scenario it is not necessary to go to the universal covering space to avoid closed timelike curves in the brane space where we live in. Since all ordinary matter is assumed to be trapped on the brane, ordinary particles and photons cannot leave

[2] Note, when x and x' are timelikely separated μ is an imaginary number.

the brane to enter orbits with closed timelike or null curves. So, the closed timelike curves in the bulk anti-de Sitter space will not bother us. (Certainly gravitons can leave the brane and enter the region with closed timelike curves.)

From Eq. (9) we obtain the ratio of the mass density in the cosmological constant to the Planck mass density in the brane universe

$$\frac{\rho_\Lambda}{\rho_{\rm P}} = \frac{f(mr_0)}{40\pi^2}\left(M_{\rm P} r_0\right)^{-6} = \frac{f(mr_0)}{40\pi^2}\left(\frac{M_{\rm P}}{m_{\rm P}}\right)^{12} , \tag{13}$$

where $\rho_\Lambda = {}^{(4)}\Lambda/8\pi m_{\rm P}^{-2}$, $\rho_{\rm P} = m_{\rm P}^4$, and $f(x) \equiv x^2\left(1+x^2/3\right)\sqrt{1+x^2/4}$. Because of the high power dependence of $\rho_\Lambda/\rho_{\rm P}$ on $M_{\rm P}/m_{\rm P}$ $[\rho_\Lambda/\rho_{\rm P} \propto (M_{\rm P}/m_{\rm P})^{12}]$, an extremely small ratio of $\rho_\Lambda/\rho_{\rm P}$ is easily generated if $M_{\rm P}/m_{\rm P} \ll 1$. For example, if $M_{\rm P}/m_{\rm P} \sim 10^{-10}$ and $f(mr_0) \sim 1$, then $\rho_\Lambda/\rho_{\rm P} \sim 10^{-123}$, in agreement with the observation value.

In Fig. 1, we plot $M_{\rm P}$ as a function of mr_0, by setting $\rho_\Lambda/\rho_{\rm P} = 10^{-123}$, the value indicated by the observations (assuming the Hubble constant $H_0 = 65\,{\rm km\,s^{-1}\,Mpc^{-1}}$). The limit given by the current gravity experiment [51] is shown with a horizontal line, with an upward arrow indicating that is a lower bound. In [51] no deviation from Newton's gravitational law was observed down to a distance of $d = 108\mu{\rm m}$. We convert this distance to a limit on r_0 by $r_0 = (3/2)^{1/2} d$ [5]. The circles on the curve in Fig. 1 correspond to the mass given by Eq (12), for $n = 1, 2, 3, 4, 5, 6$, and 10 (downward). For the case with a discrete mass spectrum, which corresponds to an anti-de Sitter space with closed timelike curves, the limit given by [51] excludes all solutions with $n \geq 5$. For the case with a continuous mass spectrum, which corresponds to the universal covering space of an anti-de Sitter space without closed timelike curves, the limit given by [51] excludes all solutions with $m > 6.3/r_0$. In both cases the solutions being excluded would produce a too large cosmological constant if the limit from the gravity experiment is satisfied. However, Fig. 1 shows that, there exist a range of solutions that are within the limit given by the gravity experiment and give rise to a cosmological constant with $\rho_\Lambda/\rho_{\rm P} \approx 10^{-123}$ in the brane universe. These solutions correspond to $n = 1, 2, 3$, or 4 for the case with a discrete mass spectrum, or $m < 6.3/r_0$ for the case with a continuous mass spectrum.

To conclude, we have shown that the vacuum polarization effect of quantum fields in an anti-de Sitter space naturally gives rise to a small but nonzero cosmological constant in a brane universe living in the anti-de Sitter space. If our four-dimensional universe is embedded in a five-dimensional anti-de Sitter space with a curvature radius $r_0 \sim 10^{-3}$cm and a fundamental Planck mass $M_{\rm P} \sim 10^9$GeV, then the stress-energy tensor arising from the vacuum polarization of a scalar field with a mass $m \sim r_0^{-1} \sim 10^{-2}$eV in the bulk anti-de Sitter space behaves like a cosmological constant in our brane universe with a value in agreement with that suggested by current cosmological observations. Probing gravity down to a scale $\sim 10^{-3}$cm (which is becoming possible [51, 52]) will provide a test for the model presented in this Letter.

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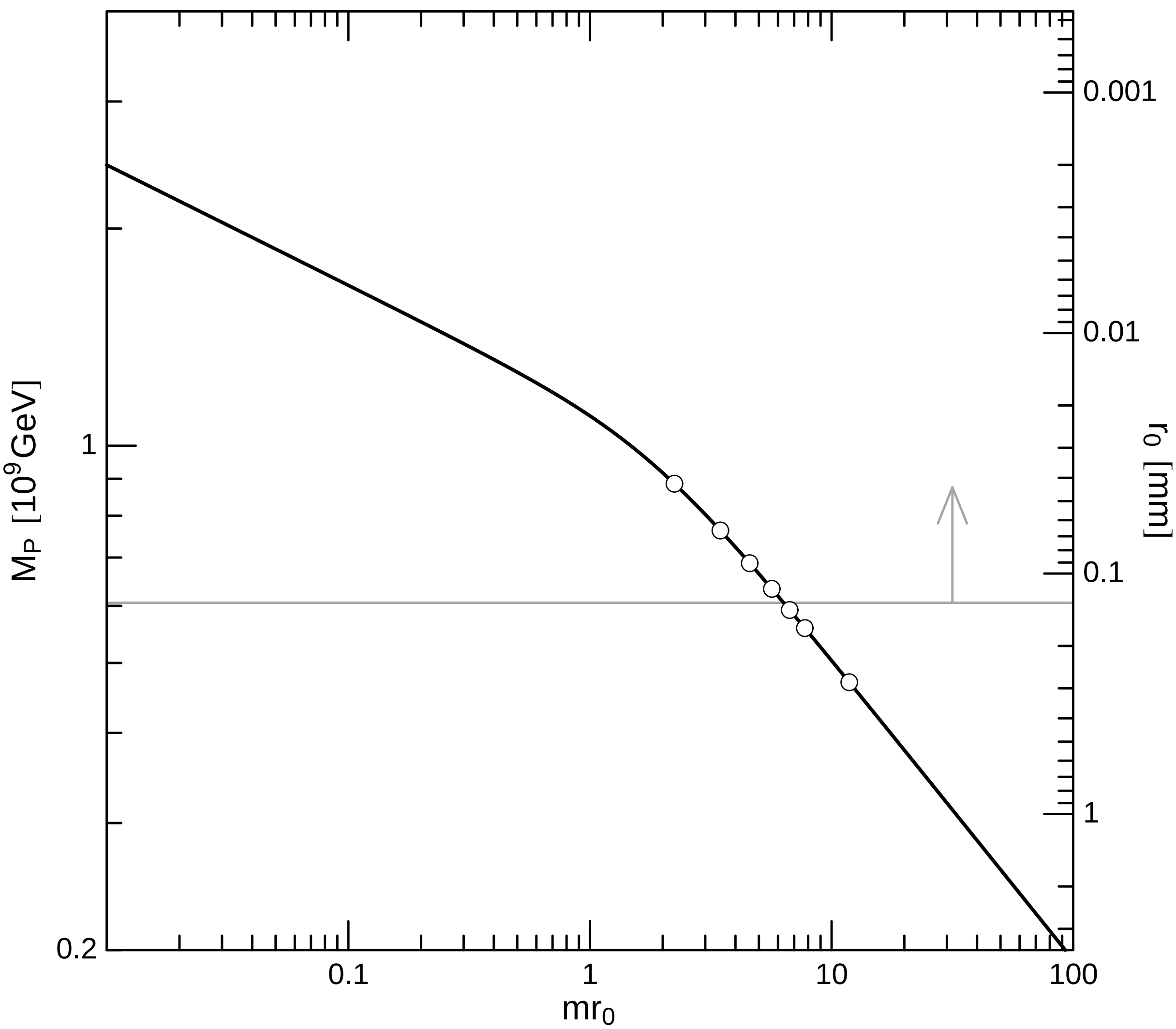

Figure 1: Plot of M_{P} as a function of mr_0, by which the ratio $\rho_\Lambda/\rho_{\mathrm{P}} \approx 10^{-123}$ is produced on the brane universe [see Eq. (13)]. The corresponding curvature radius of the bulk anti-de Sitter space, r_0 [related to M_{P} by Eq. (10)], is indicated on the right vertical axis. The horizontal line is the limit given by the gravity experiment in [51], with an upward arrow indicating that it is a lower bound. The circles on the curve show the mass m given by Eq. (12), for $n = 1, 2, 3, 4, 5, 6$, and 10 (downward).